

**Instructions: You have a total of 55 minutes to complete this test.**

**Answer each of the following questions completely.**

Time Start \_\_\_\_\_ Time finish \_\_\_\_\_ Pledged \_\_\_\_\_

**You must supply all details that led to your answer and correct SI units where required.**

**Do not discuss any aspect of this test with anyone until I return the test.**

Although you may use additional sheets of paper which should be turned in with your test, please write (neatly) your answers on the pages where the problems are presented.

**Constants:**  $e^- = -1.602 \times 10^{-19} \text{ C}$ ,  $k = 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$ ,  $\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$

[1] A wire shaped as an infinite cylinder of radius  $a$  has a current  $I$  uniformly spread across its cross sectional area.

(a) Calculate the magnitude of the current density inside the inner wire assuming the current flows along the axis.

Your answer will involve  $I$ , and  $a$ . **Be sure to show details and sketches.**

(b) Calculate the magnetic field inside the wire at a distance  $s$  ( $s < a$ ). Your answer will involve  $J$ , and  $s$ .

(c) Find the magnetic field at a distance  $s$  outside the wire. Your answer will involve  $I$  and  $s$ .

(d) If  $I=1 \text{ A}$  and  $a=.01 \text{ m}$ , provide a numerical value for  $B$  at the surface of the wire.

**[2]** A parallel plate capacitor has plates of area  $A$  and separation  $d$ . On the plate located at the origin, a surface charge density  $+\sigma$  exists while on the plate located at  $d$  a surface charge density  $-\sigma$  exists. Answer the following assuming that this can be regarded as an ideal capacitor. **Note: you must show all details (including sketches) for complete credit. Just writing answers is not enough for full credit here.**

- (a) Starting from Gauss's Law, show how to calculate the **vector** electric field within the capacitor in terms of  $\sigma, \epsilon_0$  and  $\hat{x}$ .
- (b) Calculate the **magnitude** of the potential difference between the two plates in terms of  $\sigma, \epsilon_0$  and  $d$ .
- (c) Calculate the capacitance of the capacitor in terms of  $\epsilon_0, A$ , and  $d$ .
- (d) Calculate the total energy stored on the capacitor in terms of  $C$  and  $V$ .
- (e) Calculate the energy density in terms of  $\epsilon_0$  and  $E$ .
- (f) Suppose the capacitor plates have a potential difference of 100V and the plates are separated by  $d=0.1$  m. Provide a numerical answer for the energy density together with correct SI units.
- (e) Now suppose that in some region of space the electric potential is given by:  $V = cx + ax^3$  where  $a$  and  $c$  are constants. What is the **vector** electric field in this region of space?

**[3]** A rod had a cross sectional area  $A = 4 \times 10^{-2} \text{m}^2$  and it is 3.0 m long. The material which the rod is made of has a resistivity given by  $\rho_R = 2 \times 10^{-7} \Omega\text{m}$ .

(a) Calculate the resistance of the rod.

(b) If two rods each with  $2.7 \Omega$  resistance are connected in parallel, calculate the equivalent resistance.

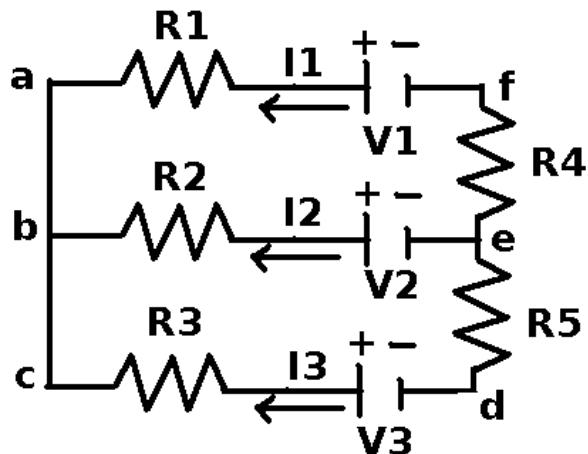
(c) If two rods each with  $2.7 \Omega$  resistance are connected in series, calculate the equivalent resistance.

A material having a dielectric constant  $\kappa = 7.5$  is inserted completely inside a capacitor. The capacitance is measured to be  $100 \mu\text{f}$  with the material inside.

(d) What is the geometrical capacitance of the system together with correct SI units?

(e) If two capacitors each with a capacitance of  $15 \mu\text{f}$  are connected in parallel, calculate the equivalent capacitance.

(f) If two capacitors each with a capacitance of  $15 \mu\text{f}$  are connected in series, calculate the equivalent capacitance.



**[4]** Consider the circuit shown above.

(a) Write down Kirchoff's loop equations for the following two loops:

(abefa): \_\_\_\_\_

(bcdeb): \_\_\_\_\_

(b) Write down Kirchoff's junction equations for the following junction:

@b: \_\_\_\_\_

Suppose that you have the following values for voltages and resistances:

$V_1=10\text{V}$ :  $V_2=20\text{V}$ :  $V_3=30\text{V}$ :  $R_1=1\Omega$ :  $R_2=2\Omega$ :  $R_3=3\Omega$ :  $R_4=4\Omega$ :  $R_5=5\Omega$ .

If the solution for the currents are:  $I_1=-1.82\text{A}$ :  $I_2=0.45\text{ A}$ :  $I_3=1.36\text{ A}$ , then:

(c) Calculate the total power dissipated in the circuit together with correct SI units.

(d) What is the interpretation of the current provided for  $I_1$ ?