	Physics 220: Worksheet 3A	Name: ________________

Supplementary problems for electrostatics

1. Three identical charges each with q=1μC are arranged at the vertices of an equilateral triangle with a side of length 1m . Find the electric field at the origin.

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

4. A sphere of radius a has a charge Q spread uniformly over its volume and at the center of the sphere is a charge –Q. Find the vector electric field inside and outside the sphere.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

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1. Three identical charges each with q=1μC are arranged at the vertices of an equilateral triangle with a side of length 1m . Find the electric field at the origin.

Align your triangle so that one charge is located at (-L/2,0), the second is at (+L/2,0) and the third is at (0,a). At the origin, the electric field in the x-direction cancels. So you only need to calculate the field along the y axis from the third charge. We need to calculate a. This is given by:

$$a = \sqrt{L^2 - \left(\tfrac{L}{2}\right)^2} = L\sqrt{1 - \tfrac{1}{4}} = L\sqrt{\tfrac{3}{4}} = \tfrac{\sqrt{3}L}{2}$$

This is:

$$\vec{r}_i = 0\hat{x} + \tfrac{\sqrt{3}L}{2}\hat{y} : \vec{r}_p = 0\hat{x} + 0\hat{y} : \vec{r}_{ip} = \vec{r}_p - \vec{r}_i = 0\hat{x} - \tfrac{\sqrt{3}L}{2}\hat{y}$$

$$\vec{E} = kq\frac{-\frac{\sqrt{3}L}{2}\hat{y}}{\left[\left(\frac{\sqrt{3}L}{2}\right)^2\right]^{3/2}} = kq\frac{-\frac{\sqrt{3}L}{2}\hat{y}}{\left[\left(\frac{3L^2}{4}\right)\right]^{3/2}} = -kq\frac{L}{L^3}\frac{\frac{\sqrt{3}}{2}}{\left[\frac{3}{4}\right]^{3/2}}\hat{y} = -\frac{kq}{L^2}\frac{0.866}{0.650}\hat{y} = -1.33\frac{kq}{L^2}\hat{y}$$

In our case at hand, we then have:

$$\vec{E} = -1.33\frac{kq}{L^2}\hat{y} = -1.196x10^4\hat{y}\,\tfrac{N}{C}$$

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.
Form each of the vectors:

$$\vec{r}_{1p} = \hat{x} + \hat{y} : \vec{r}_{2p} = -\hat{x} + \hat{y} : \vec{r}_{3p} = -\hat{x} - \hat{y} : \vec{r}_{4p} = \hat{x} - \hat{y}$$

Form the magnitudes:

$$\left|\vec{r}_{1p}\right| = \left|\vec{r}_{2p}\right| = \left|\vec{r}_{3p}\right| = \left|\vec{r}_{4p}\right| = \sqrt{2}$$

Form the unit vectors:

$$\hat{r}_{1p} = \tfrac{\hat{x}+\hat{y}}{\sqrt{2}} : \hat{r}_{2p} = \tfrac{-\hat{x}+\hat{y}}{\sqrt{2}} : \hat{r}_{3p} = \tfrac{-\hat{x}-\hat{y}}{\sqrt{2}} : \hat{r}_{4p} = \tfrac{\hat{x}-\hat{y}}{\sqrt{2}}$$

Form E:

$$\vec{E}_p = \tfrac{k\mu}{2\sqrt{2}}\left\{(1-2-3+4)\hat{x} + (1+2-3-4)\hat{y}\right\} = \tfrac{k\mu}{2\sqrt{2}}\left\{0\hat{x} - 4\hat{y}\right\} = 12713\,\tfrac{N}{C}$$

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.
In this case, only charges 1 and 2 contribute to fields along the x direction and only charge 3 contributes to the field along the y direction. The electric field vectors are easily written:

$$\vec{r}_{1p} = \hat{x} : \vec{r}_{2p} = -\hat{x} : \vec{r}_{3p} = -\vec{y} : \left|\vec{r}_{1p}\right| = \left|\vec{r}_{2p}\right| = \left|\vec{r}_{3p}\right| = 1 : \hat{r}_{1p} = \hat{x} : \hat{r}_{2p} = \hat{y} : \hat{r}_{3p} = \hat{y}$$

Then the electric field is given by:

$$\vec{E}_p = k\mu\left\{-\hat{x} - \hat{x} - \hat{y}\right\} = k\mu\left[-2\hat{x} - \hat{y}\right] = \left\{-71980\hat{x} - 8990\hat{y}\right\}\tfrac{N}{C}$$

4. A sphere of radius a has a charge Q spread uniformly over its volume and at the center of the sphere is a charge –Q. Find the vector electric field inside and outside the sphere.
Outside the sphere, since the total charge enclosed is zero, the electric flux is zero. By symmetry then the electric field outside the sphere is zero.
Inside the sphere, we have that the volume density of charge is:

$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$ so that the net positive charge enclosed is $Q_{enc} = Q\left(\frac{r}{a}\right)^3$

For a Gaussian sphere, we have the electric flux given by:

$$\Phi = EA = E\left(4\pi r^2\right)$$

So the field inside the sphere from the positive charge is given by:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r}{a^3}\right)\hat{r}$$

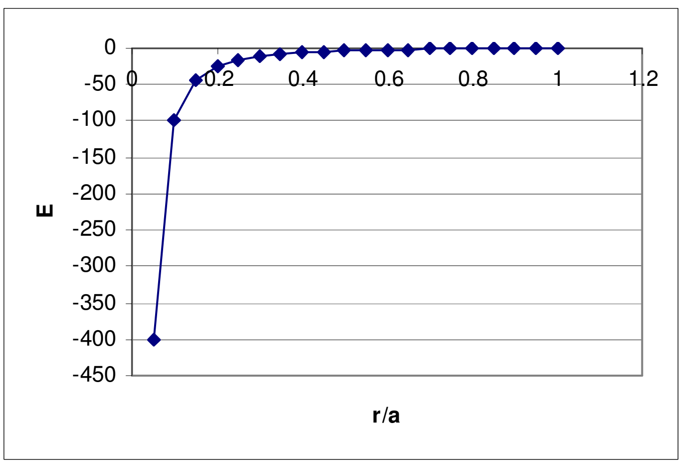
The field from the negative charge is given by

$$\vec{E} = -\frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$$

We thus find that the total electric field is given by:

$$E = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r}{a^3} - \frac{1}{r^2}\right)\hat{r}$$

A plot of the situation looks like this:



5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

The surface charge density is given by $\sigma = \frac{Q}{A} = 1\,\frac{\mu C}{m^2}$

Outside the capacitor note that the electric field is zero.
Choosing a cylindrical Gaussian surface of area A' we have that between the plates, the electric flux through the ends of the cylinder is given by:

$$EA' = \frac{\sigma A'}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{1\text{x}10^{-6}}{\left(8.85\text{x}10^{-12}\right)} = 1.13\text{x}10^5\,\frac{N}{C}$$