Energy of an electric dipole in an external field

Let's find the energy of an electric dipole in an external field. The dipole consists of two charges separated by a distance d and the positive charge is in quadrant I while the negative charge is in quadrant 3. The electric dipole vector is assumed to point from the negative charge towards the positive charge and the electric field is in the +z direction. Additionally, the angle θ is measured beginning at the +z axis. Note that the negation does not come from the angle definition.

$$W = \vec{F} \bullet \Delta\vec{z} = \left|\vec{F}\right|\left|\Delta\vec{z}\right|\cos(\theta) = \Delta U$$

$$\text{if } @\ \theta = \tfrac{\pi}{2} : U = 0 \Rightarrow \Delta U = 0 - \left|\vec{F}\right|\left|\Delta\vec{z}\right|\cos(\theta) = -qE\,\tfrac{d}{2}\cos(\theta)$$

$$2 \text{ opposite charges } \Rightarrow U = -Eqd\cos(\theta) \Rightarrow U = -\vec{p} \bullet \vec{E}$$