

**Refer to the notes about conservation of momentum before doing this worksheet.**

- (1) A mass  $m_1$  is moving with an initial velocity  $\vec{v}_1$ . The mass inelastically collides with a second mass  $m_2$  which is not moving. With what final velocity does the pair have after moving off?
- (2) A mass  $m_1$  collides elastically with a mass  $m_2$  which is initially at rest. If  $m_2 \gg m_1$ , with what velocity does  $m_1$  and  $m_2$  move off with?
- (3) A mass  $m_1$  collides elastically with a mass  $m_2$  which is initially at rest. If  $m_1 = m_2$ , with what velocity does  $m_1$  and  $m_2$  move off with?
- (4) A mass  $m_1$  collides elastically with a mass  $m_2$  which is initially at rest. If  $m_1 \gg m_2$ , with what velocity does  $m_1$  and  $m_2$  move off with?
- (5) It's a snowy day at the parking lot and two very similar cars (with the same mass) have a collision. The second car was not moving and had its breaks on and was knocked for 10 m at an angle (negative) of  $35^\circ$  from an arbitrarily placed bisector. The driver of the first car immediately applied the brakes and was diverted for 20m before stopping at an angle (positive) of  $45^\circ$  from the bisector. If the coefficient of friction between the snow and the cars is  $\mu = 0.2$ , how fast was the first car moving before the collision?

(1) A mass  $m_1$  is moving with an initial velocity  $\vec{v}_1$ . The mass inelastically collides with a second mass  $m_2$  which is not moving. With what final velocity does the pair have after moving off?

Solution: Although momentum is conserved here, kinetic energy is not since the collision is inelastic. Thus,

$$\Delta \vec{P} = 0 \Rightarrow m_1 v_{\text{before}} = (m_1 + m_2) v_{\text{after}} \Rightarrow v_{\text{after}} = \frac{m_1}{m_1 + m_2} v_{\text{before}}.$$

As a numerical example, you might consider  $m_1 = 3$  kg,  $m_2 = 5$  kg and  $v_{\text{before}} = 10$  m/s. Then:

$$v_{\text{after}} = \frac{m_1}{m_1 + m_2} v_{\text{before}} \Rightarrow v_{\text{after}} = \frac{3}{3+5} 10 = \frac{30}{8} = 3.75 \frac{\text{m}}{\text{s}}$$

(2) A mass  $m_1$  collides elastically with a mass  $m_2$  which is initially at rest. If  $m_2 \gg m_1$ , with what velocity does  $m_1$  and  $m_2$  move off with?

Solution: This is kind-of like bouncing a basket ball off of a wall. The wall doesn't move much and the basket ball comes back at you. Let's see how this comes out of our equations. We had,

$$v_{1,f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1,i} \quad \text{and} \quad v_{2,f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1,i}$$

To get the approximation here, we divide the top and the bottom of each by  $m_2$ :

$$v_{1,f} = \left( \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} \right) v_{1,i} \approx -\left( \frac{1}{1} \right) = -v_{1,i} \quad \text{and} \quad v_{2,f} = \left( \frac{2 \left( \frac{m_1}{m_2} \right)}{\frac{m_1}{m_2} + 1} \right) \approx \frac{0}{1} \approx 0$$

$$v_{1,f} \approx -v_{1,i} \quad \text{and} \quad v_{2,f} \approx 0$$

Thus, the wall stays put and the basket ball comes back at the thrower.

(3) A mass  $m_1$  collides elastically with a mass  $m_2$  which is initially at rest. If  $m_1=m_2$ , with what velocity does  $m_1$  and  $m_2$  move off with?

Solution:

This is the case of two identical balls hitting each other. What is observed is that the energy is completely transferred from the first ball to the second ball. This needs to come out of the equations also. Let's see if it does. Our equations are:

$$v_{1,f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1,i} \quad \text{and} \quad v_{2,f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1,i}$$

Now, let's let each of the masses be the same and equal to  $m$ . Then:

$$v_{1,f} = \left( \frac{m - m}{m + m} \right) v_{1,i} = 0 \quad \text{and} \quad v_{2,f} = \left( \frac{2m}{m + m} \right) v_{1,i} = v_{1,i}$$

We can then say that the result is:

$$v_{1,f} \approx 0 \quad \text{and} \quad v_{2,f} \approx v_{1,i} .$$

In other words, the first ball stops and the second ball moves off with the speed of the first ball.

(4) A mass  $m_1$  collides elastically with a mass  $m_2$  which is initially at rest. If  $m_1 \gg m_2$ , with what velocity does  $m_1$  and  $m_2$  move off with?

Solution: This is kind-of like bouncing a big bowling ball off of a ping-pong ball. We had for totally elastic collisions the following results:

$$v_{1,f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1,i} \quad \text{and} \quad v_{2,f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1,i}$$

To get the approximation, divide each of these on top and bottom by  $m_1$ :

$$v_{1,f} = \left( \frac{1 - \frac{m_2}{m_1}}{1 + \frac{m_2}{m_1}} \right) v_{1,i} \quad \text{and} \quad v_{2,f} = \left( \frac{2}{1 + \frac{m_2}{m_1}} \right) v_{1,i}$$

Now, let the ratios become about equal to zero. This gives the surprising results:

$$v_{1,f} \approx v_{1,i} \quad \text{and} \quad v_{2,f} \approx 2v_{1,i}$$

(5) It's a snowy day at the parking lot and two very similar cars (with the same mass) have a collision. The second car was not moving and had its breaks on and was knocked for 10 m at an angle (negative) of  $35^\circ$  from an arbitrarily placed bisector. The driver of the first car immediately applied the brakes and was diverted for 20m before stopping at an angle (positive) of  $45^\circ$  from the bisector. If the coefficient of friction between the snow and the cars is  $\mu=0.2$ , how fast was the first car moving before the collision?

Solution:

We can find out how fast the second car was traveling immediately after the collision:

$$\Delta K_{NC} = \Delta K_C \Rightarrow -\mu mgx = -\frac{1}{2}mv^2 \Rightarrow v = \sqrt{2\mu gx}$$

where I have assume a positive velocity. We can now answer questions about the initial velocity of the first car by finding the components of the final velocities. **We only really know for sure, in this collision, that the momentum was conserved at the moment of impact.**

For this situation, the velocity of the second car right after the collision was 6.26 m/s.

$$\text{How? } v = \sqrt{2\mu gx} = \sqrt{2(.2)(9.8)(10)} = 6.261 \frac{\text{m}}{\text{s}} .$$

The momentum components for the second car are:

$$P_{x,2} = m_2 v_{x,2} = m_2 (6.26) \cos(-35) = 5.13 m_2$$

$$P_{y,2} = m_2 v_{y,2} = m_2 (6.26) \sin(-35) = -3.59 m_2$$

We can, in the same way, find out how fast the first car was moving after the collision, which is 8.85 m/s.

$$\text{How? } v = \sqrt{(2\mu gx)} = \sqrt{2(.2)(9.8)(20)} = 8.854 \text{ m/s} .$$

The components of momentum for the first car are:

$$P_{x,1} = m_1 v_{x,1} = m_1 (8.85) \cos(45) = 6.25 m_1$$

$$P_{y,1} = m_1 v_{y,1} = m_1 (8.85) \sin(45) = 6.25 m_1$$

Since momentum is conserved, we then have:

$$P_{x,1,\text{before}} = P_{x,1,\text{after}} + P_{x,2,\text{after}}$$

$$P_{y,1,\text{before}} = P_{y,1,\text{after}} + P_{y,2,\text{after}}$$

Since the masses are the same, we thus have:

$$v_{x,1,\text{before}} = 6.25 + 5.13 = 11.38 \text{ m/s}$$

$$v_{y,1,\text{before}} = 6.25 - 3.58 = 2.66 \text{ m/s}$$

We can now answer a question about how fast the first car was moving:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{11.38^2 + 2.66^2} = 11.69 \text{ m/s} .$$

If you decide to become an accident investigator, however, before you go to court make sure you've checked the kinetic energy of the situation or you'll have an unpleasant time in court. One of the morals to this story is that not all collisions are elastic or inelastic and know your physics before you become an expert witness. Be aware, this is the simplest of all possibilities. Many other things happen in an actual accident that need to be considered.