Archimedes' Principle and pressure Pandemic Edition Fall 2020

Archimedes' principle states that the buoyant force is equal to the weight of the fluid displaced. In equation form, we write this as:

$$F_b = W_{\text{of fluid displaced}} \quad .$$

For an object which displaces a volume V, the weight of the fluid displaced is given by:

$$W = \rho_{\text{fluid}} V g \quad ,$$

where ρ_{fluid} is the density of the fluid that the object is placed in. In the case of water, $\rho_{\text{water}} = 1x10^3 \frac{kg}{m^3}$. V is the volume **of the displaced fluid** and g is the acceleration due to gravity. If the object is completely submerged in the fluid, then

$$V = V_{\text{object}} \quad .$$

Let's examine the two obvious cases for an object (such as a metal cylinder) completely immersed in water and then for a block of wood floating on water. In the case of the cylinder, we find that the buoyant force is given by

$$F_b = \rho_{\text{water}} g V_{\text{object}} \quad .$$

Now, what will happen for the metal cylinder is that we would see the apparent weight decrease by the buoyant force or

$$\Delta \text{Weight} = g V_{\text{cyliner}} \left(\rho_{\text{cyliner}} - \rho_{\text{water}} \right) \quad .$$

In the case of the wooden block, since the block floats on water, we know immediately that the density of the wooden block is less than that of water. We can answer a different question then for the wooden block, namely how much of the wooden block would be immersed. This fraction, (which is a percentage when multiplied by 100) is obtained by equating the buoyant force to the weight of the block, or

$$F_b = W_{\text{object}} \quad .$$

Again, this is true for a floating object.

We can rewrite this in terms of volumes and densities as

$$F_b = \rho_{fluid}\, g\, V_{fluid\ displaced} = \rho_{fluid}\, g\, V_{object\ immersed}$$

where

$$W_{object} = \rho_{object}\, g\, V_{object} \quad .$$

We solve this for the fraction of the object immersed, we find that this is given by:

$$\text{Floats} \Rightarrow F_b = W_{oject}$$
$$\Rightarrow \rho_{fluid}\, g\, V_{fluid\ displaced} = \rho_{oject}\, g\, V_{object}$$
$$\Rightarrow \rho_{fluid}\, V_{fluid\ displaced} = \rho_{object}\, V_{object}$$
$$\Rightarrow \frac{V_{object\ immersed}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}$$

The Procedure
Experiment 1

Choose the mass of the spheres in the first simulation. You will need to solve (for the simulation) a cubic equation approximately. This is so you can determine where nature will decide to float the sphere. In lab, I will show you how to do that. Then you will change the water level until the to of the sphere is level with the top of the graduated cylinder. in the first part of the lab, you will use the result that the cap of a sphere is related to the height by:

$$V_{cap} = \frac{1}{3}\pi h^2 (3R - h)$$

Now I do want to point out that in the simulation, the graduated cylinder is huge. It is about 1 meter tall and the reason I use a sphere in these simulations is that I can effectively reduce the problem to a one dimensional problem by doing this. And it was easier for me. Also the sphere inside the cylinder is also quite large. I advise do not resize the plot of the graduated cylinder: doing so will really mess up the very careful calibration I made on the sphere size.

Your simulation will produce two plots after you take your data (do not choose my masses). The first plot will provide verification of both your measurements and your solutions to the required cubic equation. It should result in a straight line with small deviations that is related to the curvature of the sphere. The deviations are a subtle source of error.

In the second plot, I plot the measured buoyant force from your measurements of the volume of the sphere displaced in the water. You should choose data starting at 40 and ending at 60. I have also provided a linear fit to your data. If the buoyant force is equal to the weight of the fluid displaced, then you should have a straight line. This is verification of Archimedes' principle.

So in summary you will pick a mass to start with. Then solve the cubic equation approximately for the root closest to the left end of the control tool. The root is found when the sign of the error switches sign. Be careful not to choose one of the higher roots here: there are 3 solutions to the cubic equation and the general solution is quite nasty which is why you solve it here. If your fall diappears, you have picked the wrong solution. The 3rd step to make your measurement is to change the water level until the ball is level with the top. Copy and paste special the 4 measurements . Increment your mass and repeat until you have completed your chart. You will notice that the intercept of your fit equation shows a zero intercept. This is interpreted as when no volume is immersed, the buoyant force is zero. You will also note that the slope of the graph is given by:

$$\frac{\Delta F_b}{\Delta V_{submerged}} = \rho_{water} g$$

If the sphere were floated on another fluid, especially with one with a density greater than water, it would float higher. An example of this is found when I swim in the mediterrian sea: it is difficult to get into a drowning situation because I seem to float too good for this to happen. Don't try my experiment at home.

Also if you look closely at the deviations in the other graph, in fact they really do reproduce the curvature of the sphere as the sphere sinks lower and lower in the fluid pretty much. The curvature is not linearly related to the depth of the sphere submerged. If your experiment is done good enough, it should be fairly clear. You may have to zoom in on the ball for a better measurement.

Experiment 2

For experiment 2, you will fill the graduated cylinder and measure the relationship between the immersion of a sphere with a higher density than that of water. You will choose the mass of the sphere (don't choose mine) and measure the effective weight as a function of the sphere as a function of the amount of water displaced. The indicator on the spring scale should not be used to make this measurement. It is eye candy.

Your procedure here is to pick a mass of the sphere. Don't choose my mass. then raise the water level to 40 cm. Watching the video you will note that I did not precisely reproduce the physics of changing the water level here exactly. That would have been real hard to do.

In one graph I plot water level vs immersed weight. Provided you spread your data out, you will again see the deviations owing to the fact that the sphere is not linerally related to the depth immersed in the fluid (and the curvature is first one way and then the other). I really believe when I learn things from my simulations, such as this, it is a good simulation.

You will increase the water level up to 60 cm, recording the 3 variables by cut and paste special number in the area indicated. In the second graph you will notice that you have an intersection in your linear fit. You should look through your parameters (in orange at the top) and interpret this and explain it. You will also notice the 9800 slope. From experiment 1, you know how to interpret this.

Sample Calculations

Ice has a density of $\rho = 975\,kg/m^3$ while sea water has a density of about 1100 kg/m^3. What percentage of the ice is below the water. How does this relate to the expression "this is only the tip of the ice berg?"

An object floats with 50% of it's volume submerged in fluid 1. The same object floats with 75 % of its volume submerged in fluid 2. If the first fluid is water, what is the density of the second fluid?

Experiment 4

This is not done in today's lab, but I provide it for you to read purely for your interest. Normally this is done in the physical lab experience.

You are now going to verify Archimedes' principle for a helium balloon. Your results will have some error since the helium in the balloon is not pure.

Show me the recorded weight of your balloon and I will fill it with helium. We will measure the pressure using the simple water manometer that I have. The over pressure is given from the difference in levels (in m) of the water in the manometer.

$$\Delta P=\rho_{water}\,g[\Delta h]=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ \text{Pa}$$

Thus the absolute pressure inside the balloon is given by the

$$P=P_{atm}+\Delta P=101325+\Delta P=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ \text{Pa}$$

Note that in a more precise experiment, we would measure the atmospheric pressure.

Now tie a short thread to the balloon and determine the volume of the inflated balloon. This is done by placing the balloon into a box of peanuts. Fill the box level with the top by placing peanuts into the box. Remove the balloon. The empty part of the box is equal to the volume of the balloon. The volume of the balloon is given by the length x width x height of this **empty part** of the box.

$$V_{balloon}=\text{Length x Width x Depth}=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ m^3$$

Interestingly enough, you will also need to record the temperature in C which must be converted to K (T[K]=273.15+T[C]).

$$T=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ C=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ K$$

We can now calculate the number of molecules of helium in the balloon by using the ideal gas equation of state.

$$N=[P+\Delta P]\frac{V_{balloon}}{kT}=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ \text{molecules}$$

where k is Boltzman's constant and is given by $k=1.3806503\text{x}10^{-23}\frac{\text{Joules}}{K}$.

In order to find the mass of the helium inside the balloon, we first find the number of moles inside the balloon. This is given by

$$n=\frac{N}{N_A}=\frac{N}{6.02\text{x}10^{23}}=\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\ \text{moles}$$

Assuming the gas inside the balloon is only helium, we can then obtain the **total mass** of the helium inside the balloon:

m=4n (where 4 is the atomic mass of helium).

So m=________________g=______________________ kg

Now tie a weight hanger to the end of the thread and place about 20 g mass on the weight hanger. Weight the mass of the hanger and then allow the balloon to lift on the mass and measure it again. The difference in the masses is how much mass the balloon can lift. Record this value.

$$\Delta m = Mass_{\text{hanger w/o balloon}} - Mass_{\text{hanger with balloon}} = \_\_\_\_\_\_\_\_\_\_\_\_ \text{ g} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ kg}$$

The total amount of weight which the balloon can lift is then given by:

$$lift = [m_{balloon} + m_{helium} + \Delta m] g = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ N}$$

Now we can determine the theoretical lift from Archimedes' principle. We will assume the density of air is about 1.2041 kg/m^3, although this is only approximate. In a more precise experiment, we would measure this also.

The theoretical buoyant force would then be:

$$F_b = \rho_{air} V_{balloon} g = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{ N}$$

Compare this to the measured lift:

$$\%difference = 100 \text{ x } \frac{F_b - lift}{\frac{1}{2}[F_b + lift]} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Your writeup for this lab consists of the normal abstract, procedures, this data sheet filled in with the results of each of the calculations for this lab which are contained in the spreadsheet and conclusions with references. Oh, and yes you can keep your balloon!