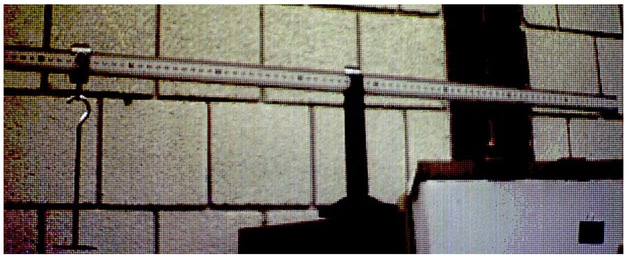
Pandemic Equilibrium of Rigid Bodies



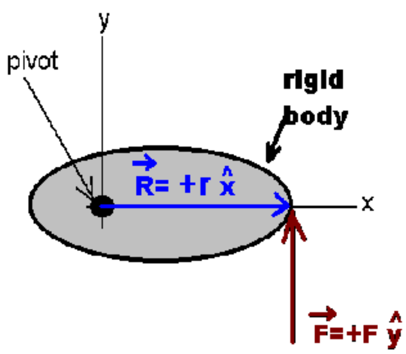
In this lab, we consider some of the consequences and conditions for static equilibrium. **The conditions for static mechanical equilibrium for a rigid body are**

$$\sum \vec{F} = \vec{0} \quad \text{and} \quad \sum \vec{\Gamma} = \vec{0}.$$

You are, by now, quite familiar with the first of these conditions and have worked with this in an earlier lab or two. The second condition needs some explanation.

A torque (Γ or τ is the usual symbol used to represent this) is a force acting at a distance on a body. The torque is a vector which comes about in an unusual manner (and is actually a different type of vector). The torque acting on a body is defined as the cross product of the radius at which the force is acting and the force which is acting:

$$\vec{\Gamma} = \vec{R} \mathbf{X} \vec{F} \quad .$$



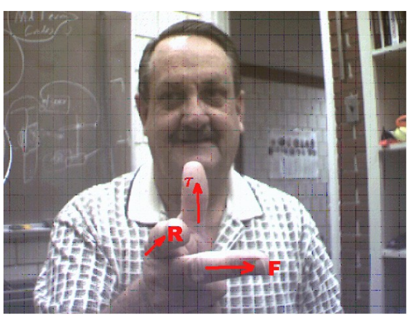
Let's consider a simple situation. Suppose the force is acting upon a body at a distance r from a pivot point as shown below. In this simple example, $\vec{F} = |\vec{F}|\hat{j}$ and $\vec{R} = |\vec{R}|\hat{i}$. The torque can then be calculated by using the determinate:

$$\vec{R} \mathbf{X} \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ R & 0 & 0 \\ 0 & F & 0 \end{vmatrix} = \hat{i}(0) + \hat{j}(0) + \hat{k}(RF) \quad .$$

In this simple example, the torque would be given by

$$\vec{\Gamma} = RF\hat{k} \quad .$$

We can also determine the magnitude of the torque by the definition of the cross product. The magnitude of the cross product of two vectors $\vec{A}$ and $\vec{B}$ is given by



$$|\vec{A} \mathbf{X} \vec{B}| = |\vec{A}||\vec{B}|\sin(\theta)$$

where θ is the angle between vectors $\vec{A}$ and $\vec{B}$. Interestingly enough, we can use this definition together with the cross product which comes by taking the determinate of the two vectors to obtain the angle between the two vectors: $\sin(\theta) = \frac{\vec{A} X \vec{B}}{|\vec{A}||\vec{B}|}$. This can be combined with the dot product to give:

$$\tan(\theta) = \frac{|\vec{A} X \vec{B}|}{\vec{A} \cdot \vec{B}} \quad .$$

There are many ways that have been developed to obtain the cross product using your right hand (these rules are known as "right hand rules"). Essentially, if you align your forefinger along the direction of R, and point your middle finger along the direction of F then your thumb points along the direction of the torque as shown. Check this with the previous example: align F along +y, R along +x then the torque will lie along +z. Ultimately these come about because our math uses a right-handed coordinate system in which $\hat{x} \mathbf{X} \hat{y} = \hat{z}$ and the requirement that the volume of the elementary parallelepiped which is given by $\text{volume} = (\hat{x} \mathbf{X} \hat{y}) \bullet \hat{z}$ be a positive quantity.

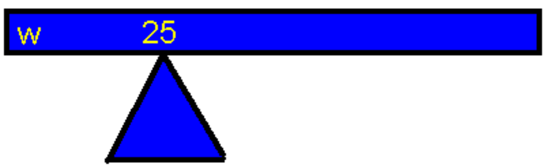
Terminology Note

Important note on my terminology: pivot point is the point that something like a pivot is touching a body (it would, for example, be where the point of the triangle below touches the meter stick) and is a physical object in touch with another physical object. Axis is an arbitrary location about which torques are located. It can also be at the pivot point but this is not a necessity.

Sign Convention for torques

We also use a convention for the sign of the torque. If a torque by itself would make a rotation in the clockwise direction about an axis, this is a negative torque. If a torque by itself would give a counterclockwise rotation about an axis, this is a positive torque. As you now know from the previous discussion, the positive or negative only refers to whether the torque lies along the positive z axis or the negative z axis. ***In addition, a positive torque about one axis may become a negative torque about another axis.***

Read the previous sentence again please!



An example: Let's do an example to show that in equilibrium, the conditions for equilibrium are met. Consider a meter stick which is pivoted at the 25 cm mark which weighs 0.1 kg. Where would a 0.5 kg weight need to be placed in order for the meter stick to balance on the pivot?

If the meter stick weighs 0.1 kg, then since it is in static equilibrium, we require $\sum \vec{F}_{ext} = \vec{0}$. . We first find the force exerted by the pivot on the meter stick.

$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow F_{pivot} - m_1 g - m_2 g = 0 \Rightarrow F_{pivot} = (m_1 + m_2) g \quad .$$

In this example, this force is given by:

$$F_{pivot} = (0.1 + 0.5) 9.8 = 5.88\,N \quad .$$

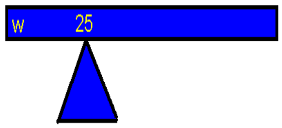
Now calculate the sum of the external torques acting on the meter stick about an axis. I am going to choose the pivot point to be my axis although this is not a necessary choice if the meter stick is in equilibrium.

The torques that you have here are the following:

(1) The weight of the meter stick acts at the center of mass of the meter stick.
(2) The 0.5 kg weight acts at the position that it is placed.
(3) The pivot acts at the pivot point.

Students will often neglect the torque contributed by the pivot point and do not understand why the answers to not provide a total external torque of zero. Students also often neglect the center of mass of a meter stick. Don't be one of these students.

Read the previous three sentences again!



(1) The pivot was placed at x=0.25 m. The center of mass of the meter stick is at x=0.5 m. Thus, regarding the axis as being at the pivot point, we see that the torque from the meter stick is acting at a distance of 0.25 m from the axis.

(a) Looking at the picture, draw in this force and show its distance from the axis.
(b) Now ask yourself the question: does this produce a clockwise or a counter clockwise rotation about the axis? The answer is clockwise. Thus this torque is negative.
(c) The value of the torque is then given by:

$$\Gamma=-(0.25\text{m})(0.1\text{kg})(9.8\frac{\text{m}}{\text{s}^2})=-0.245\,\text{NM}.$$

(2) The 0.5 kg weight acts at some (unknown) x position. It is at a distance of $[x-0.25\,\text{m}]$ away from the axis.
(a) Looking at the picture, draw in this force and show its distance from the axis. Use your intuition in making this sketch. It does not need to be perfectly located but it does need to be on the proper side of your pivot.
(b) Now ask yourself the question: does this produce a clockwise or a counter clockwise rotation about the axis? The answer, if you have done things correctly is counter clockwise. Thus this torque is positive.
(c) The value of the torque is then given by:

$$\Gamma=+(|x-.25|\text{m})(0.5\text{kg})(9.8\frac{\text{m}}{\text{s}^2})=+4.9|x-.25|\text{Nm}.$$

(I've used absolute value signs to make sure things stay of the proper sign)

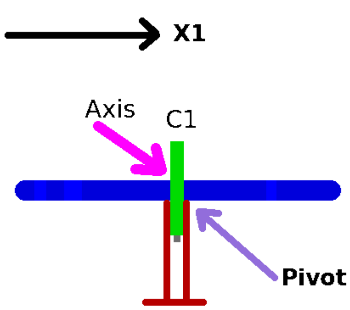
(3) The pivot exerts an upward force on the meter stick given by 5.88N. It is at a distance of 0 m from the axis.
(a) looking at the picture, draw in this force and show its distance from the axis.
(b) Now ask yourself the question about the sign of this torque. (here the answer is that it is neither positive nor negative. (see (c) below).
(c) The value of this torque is zero since it is acting through the axis.

Now solve for the position:

$$\sum\Gamma=0\Rightarrow-0.245\text{Nm}+4.9|x-.25|\text{Nm}+0\text{Nm}=0\Rightarrow x-0.25=0.05\Rightarrow x=0.2\,\text{m}.$$

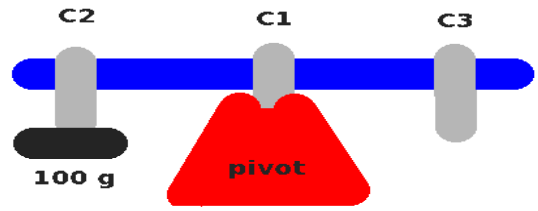
So you would need to place the weight at a distance of 0.2 m away from the pivot.

The Experiments
Experiment 0



We are going to start with experiment 0 on the static equilibrium simulation. To do this balance the meter stick. In this situation, the center of mass of the meter stick is right over the axis and pivot. Do this in simulation 0 now. You should imagine that there is also a clamp attached to the meter stick at the center of mass of the meter stick. For the simulation, the values of the meter stick and the clamp are provided from previous lab data. You should change the pivot until the meter stick is balanced. I have already filled in the signs of the torques since the signs here are all zero. Look in row 15 here. The torques are identified as: τ_1 comes from c_1 and τ_4 comes from the center of mass of the meter stick and τ_5 comes from the pivot force. The measured center of mass of the meter stick will be used for all the following experiments. I am providing the mass of the meter stick and the clamp C1. This clamp has no hanger attached.

Experiment 1:



Two other clamps are now attached to the system in experiment 0. Weigh each of the clamps and record the weight of each. I am providing these weights from previous data. So you will not need to change them.

Leave C1 at the position you determined in experiment 0. Note that **the screws point downward** (our simulated clamps have the screws pointed down). This makes it a stable system since the "y" center of mass is below the pivot. Weigh two weight hangers and record their weights (I am providing these values).

Place clamp (C2) at the 15-cm position [X_2=0.15] (15 cm from 0) with the screw pointing downward. Place on (C2) a mass hanger [H2] and a mass [M2]=100g. Record the actual weights on C2. Each of these values I have provided for you.

Place clamp (C3) on the meter stick, together with a hanger [H3] and a mass [M3] of 200g. I have provided these values for you.

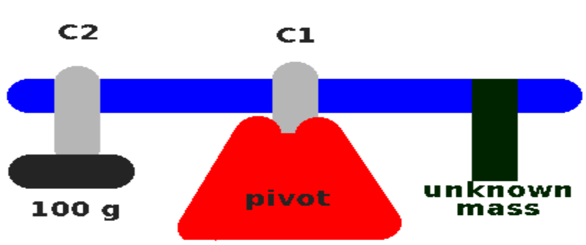
Move (C3) until you establish the condition for equilibrium with the control bar. There may be a small range here so copy the value in F21 and paste special in the appropriate cells H21, H22. The simulation will average these values.

In the simulation for Experiment 1, I have set the signs of the torques so that you can see the signs and use this knowledge for the rest of the experiments. Change the signs and see the results. It is essential that you understand how the signs of the torques are determined.

Compute the torques about the pivot point and find the percent deviation in the computed values (this means, add up all your positive torques(+τ) and your negative torques(-τ).) You will want to use the formula for the percent deviation below:

$$\%\ \text{deviation} = 100\text{x}\frac{|\tau_+|-|\tau_-|}{1/2\left(|\tau_+|+|\tau_-|\right)} \quad .$$

Experiment 2:



Consider your unknown mass to be the mass hanger, the meter stick clamp (C3) and unknown mass which is attached. Change the position of your unknown mass until equilibrium is established. Use the condition for equilibrium to determine the mass of your unknown mass. Now you will weigh your unknown mass by requiring the sum of external torques be minimized. Do this by changing the purple control. The signs here are the same as in experiment 1. When you minimize the torques, the unknown mass will appear and a %error will be calculated. The % error will be pretty good. It's a simulation. In the home experiment, you will encounter a more appropriate result.

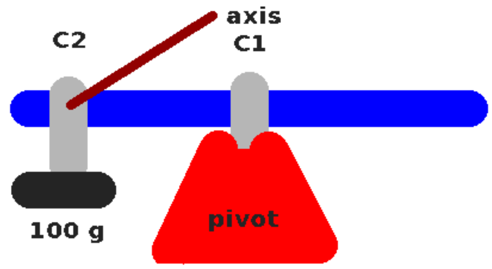
After this:

Weigh your unknown mass (which included clamp C3, the hanger and the ring stand clamp) and determine the percentage error in your measurement. **In the simulation, I have already provided this for you.**

Note: your experimental setup is exactly the same for experiment 3, 4a and 4b. Also note that the #div/0! errors will go away as you put the signs in.

Experiment 3:

Remove all clamps, masses and hangers except for C1. Place a clamp C2 on your meter stick near the end. Place hanger H2 on C2 and place a mass M2=100 g on the hanger. I am providing these values for you today from previous data.

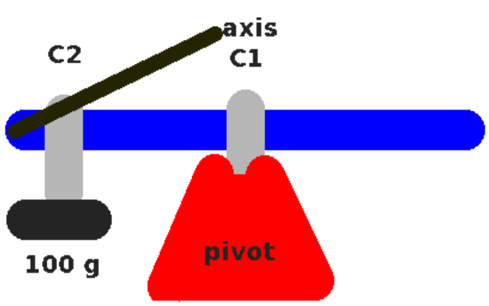


Regard your axis (but not your pivot) as being at the position of clamp C2. I have removed the signs of the torques here. So you will need to look at the image and put in the appropriate signs. This is absolutely essential for the rest of the lab to work properly. For the rest of this I will assume you have the torques about the axis properly signed.

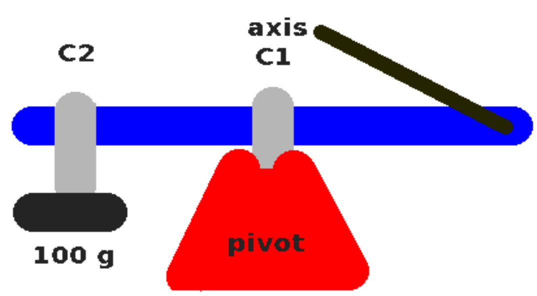
Move the meter stick in the support clamp (C1) until the system is in equilibrium using the purple control.

Experiments 4a and 4b

Leave your system in equilibrium as in Experiment 3.
Experiment 4a: the axis is at the left end of the meter stick (0 m).
Experiment 4b: the axis is at the right end of the meter stick (1 m).



Experiment 4a.



Experiment 4b.

As in experiment 3, **you will need to make absolutely sure you can obtain the correct signs with your torque**. Now, think about this: comparing experiment 3, 4a and 4b, you may not see the same deviations result. This is because of small round off errors in limiting the precision of your work.