

A string of length 2 m has a mass per unit length $\mu=0.5\text{ kg/m}$ and is under a tension $T=200\text{ N}$. Find and sketch the two lowest modes of oscillation when the system has (1) fixed boundary conditions and (2) free boundary conditions.

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$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.5}} = 20\text{ m/s}$$

$$\text{fixed: } \frac{1}{2}\lambda_1 = L \Rightarrow \lambda_1 = 2L : f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{20}{4} = 5\text{ Hz} : \lambda_2 = L \Rightarrow f_2 = \frac{v}{L} = \frac{20}{2} = 10\text{ Hz}$$

$$\text{mixed: } \frac{1}{4}\lambda_1 = L \Rightarrow \lambda_1 = \frac{v}{4L} : f_1 = \frac{v}{\lambda_1} = \frac{20}{8} = 2.5\text{ Hz}$$

$$\frac{3}{4}\lambda_3 = L \Rightarrow \lambda_3 = \frac{4L}{3} \Rightarrow f_3 = 3 \frac{v}{4L} = \frac{60}{8} = 7.5\text{ Hz}$$