

A mass m_1 collides and sticks to a mass m_2 . After the collision, the mass combination is seen to move with a speed v .

(a) If mass m_2 was at rest before the collision, how fast was mass m_1 moving before the collision in terms of m_1 , m_2 and v ?

(b) What fraction of the initial kinetic energy was lost in this collision in terms of m_1 , m_2 and v ?

(c) Suppose the mass combination encounters a frictionless inclined plane with a moderate angle of inclination. How far above the base of the plane will the mass combination be at the instant it stops in terms of v and g ?

(d) If m_1 acquired its original momentum from the decompression of a spring of spring constant k , how much was the spring compressed in terms of k , m_1 , m_2 and v ?

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(a) If mass m_2 was at rest before the collision, how fast was mass m_1 moving before the collision in terms of m_1 , m_2 and v ?

Assuming that the collision is totally inelastic, we have:

$$\Delta \vec{P} = \vec{0} \Rightarrow m_1 v_{1,b} + m_2 v_{2,b} = (m_1 + m_2) v_A : v_{2,b} = 0 \Rightarrow v_{1,b} = \frac{m_1 + m_2}{m_1} v$$

(b) What fraction of the initial kinetic energy was lost in this collision in terms of m_1 , m_2 and v ?

For a totally inelastic collision,

$$\begin{aligned} \text{fraction lost} &= 1 - \frac{K_f}{K_i} \\ &\Rightarrow 1 - \frac{\frac{1}{2}(m_1 + m_2)v^2}{\frac{1}{2}m_1 v_{1,b}^2} = 1 - \frac{(m_1 + m_2)v^2}{m_1 \left(\frac{m_1 + m_2}{m_1}\right)^2 v^2} = 1 - \frac{m_1}{m_1 + m_2} = \frac{m_1 + m_2}{m_1 + m_2} - \frac{m_1}{m_1 + m_2} \\ &\Rightarrow \text{fraction lost} = \frac{m_2}{m_1 + m_2} : \% \text{ lost} = 100 \times \frac{m_2}{m_1 + m_2} \end{aligned}$$

In particular, note that if $m_2 = 0$ then no kinetic energy is lost.

(c) Suppose the mass combination encounters a frictionless inclined plane with a moderate angle of inclination. How far above the base of the plane will the mass combination be at the instant it stops in terms of v and g ?

Apply energy principles here:

$$\begin{aligned} \Delta K_{NC} &= \Delta K_C + \Delta U \\ \Delta K_{NC} &= 0 \\ \Delta K_C &= K_f - K_i = 0 - \frac{1}{2}(m_1 + m_2)v^2 = -\frac{1}{2}(m_1 + m_2)v^2 \\ \Delta U &= U_f - U_i = (m_1 + m_2)gy - 0 = +(m_1 + m_2)gy \\ \Rightarrow 0 &= -\frac{1}{2}(m_1 + m_2)v^2 + (m_1 + m_2)gy \Rightarrow y = \frac{v^2}{2g} \end{aligned}$$

(d) If m_1 acquired its original momentum from the decompression of a spring of spring constant k , how much was the spring compressed in terms of k , m_1 , m_2 and v ?

$$\begin{aligned} \Delta K_{NC} &= \Delta K_C + \Delta U \\ \Delta K_{NC} = 0 : \Delta K_C &= \frac{1}{2}m_1 v_{1,b}^2 : \Delta U = \Delta U_s = -\frac{1}{2}kx^2 \Rightarrow 0 = \frac{1}{2}m_1 v_{1,b}^2 - \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{m_1}{k}} v_{1,b} \\ x &= \sqrt{\frac{m_1}{k}} \frac{(m_1 + m_2)}{m_1} v = \sqrt{\frac{(m_1 + m_2)^2}{km_1}} v \end{aligned}$$