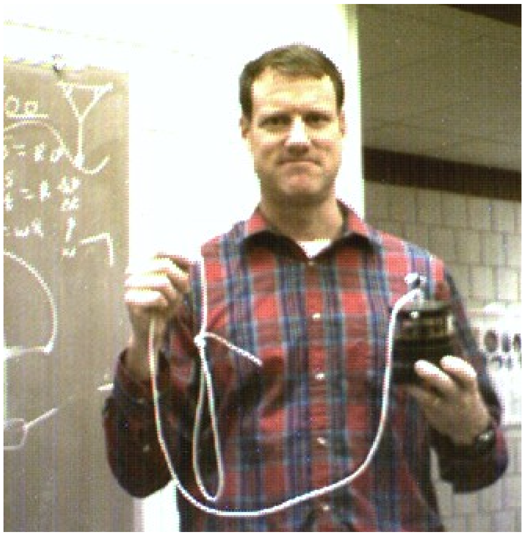
Standing Waves & Vibrations
Revised for 2013 b



You may recall from class that we developed a method for determination of the resonant frequencies of oscillation for transverse waves on a string. Here, let's summarize the method.

Suppose a wire is stretched between two rigidly fixed points. The wire has a length L. To find the modes of oscillation which exist on this system, assume the tension in the string is T and the string has a mass per unit length of μ. The speed of a transverse linear wave propagating on a medium such as a string is then given by

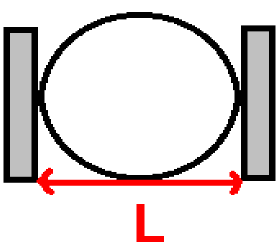
$$v=\sqrt{\frac{T}{\mu}} \quad .$$

The wavelength, frequency and speed for any wave which is linear (and non-lossy) is given by

$$f\lambda=v \quad .$$

(Important note: do not confuse the symbols T (period) and T (tension))

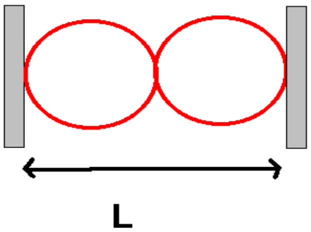
The v which appears here is the wave speed. Since our waves are all liner, v does not depend upon frequency or wavelength, but only upon tension and linear mass density. Thus, for all modes of oscillation which exist on a string, so long as the excitations remain linear, v is the same. However, because in today's lab we will be changing the tension, in fact v will change for the different tensions.



Let us impose the condition of 2 fixed boundary conditions upon a vibrating string, as shown in the following sketches. Further, it is worth your while to note that if you have a node, this is essentially a fixed boundary condition.

Since the frequency of wavelength is related by $f\lambda=v$, and for the lowest lying mode, you know that $\lambda=2L$ since for this waveform, you have nodes at each wall. Thus, the lowest lying frequency is given by

$$f_1=\frac{v}{2L} \quad .$$



If we want to find the next highest mode of oscillation, we find that 1 wavelength fits into the length L, so $\lambda=L$. This allows us to find the frequency of oscillation for this mode to be

$$f_2=\frac{v}{L}=2f_1 \quad ..$$

It is now easy to show that the nth harmonic will be given by

$$f_n=n\,f_1 ; n=1,2,3,...$$

This is all well and good if you can vary the frequency of an oscillator which is exciting the string. In principle, we could also and it will only cost a ridiculous amount of money per set-up (it involves buying bipolar amplifiers from our favorite source). Instead, in today's lab, we will leave the frequency of oscillation set at 60 Hz, and allow several modes to arise on the string. In each case, we will thus have f=60 Hz but by varying the tension and the mass in the string, we will be able to verify the following connections:

$$(1) \quad f\lambda = v$$

$$(2) \quad v = \sqrt{\frac{T}{\mu}}$$

In no event in today's lab should you place more than 250 g tension on your string, otherwise the oscillator will be deformed permanently.

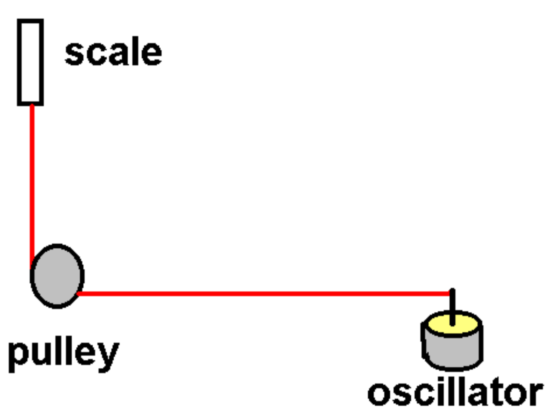
We will do the analysis for Experiment 1 and Experiment 2 in two ways in today's lab. First, using the calculated speed (2 above) we will find the experimental frequency and compare it to 60 Hz. The second manner is a graphical analysis which is as follows:
Using equation (1) and (2) we have:

$$f^2\lambda^2 = v^2 = \frac{T}{\mu} \;\Rightarrow\; \lambda^2 = \frac{1}{f^2\mu}T$$

So if we plot tension vs wavelength squared, a linear fit would provide us with the slope which then would provide us with a second experimental frequency:

$$f_{experimental} = \sqrt{\frac{1}{\mu \, x \, slope}}$$

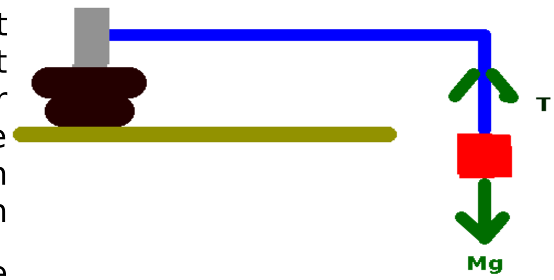
This can then be compared to the actual frequency of 60 Hz. The second method of analysis is more pure because it is not assuming as much as the first method however the average of the frequencies obtained in the first method will probably produce better results. This is due, in part, to the fact that the mass per unit length will change slightly as the tension is increased. This will have a larger effect on the slope than it will for the averages.



Experiment 1. Using the gram-spring scale connected to one end of the fat string separate at least 1 meter distance between the oscillator and the pulley. You will need to measure the mass of your string and also the length of it.

Turn on the oscillator and move the pulley until the string is slightly slack. Slowly move the pulley away from the oscillator until a standing wave pattern is observed. Measure the scale reading the length of ½ of the wavelength of this standing wave pattern. Continue to move the pulley away from the oscillator and you will obtain a total of 6 measurement locations. Record the tensions and the length of ½ of the wavelength for each of these patterns. You will want to choose a wave closer to the pulley since the oscillator does not provide us with a true fixed boundary condition. Calculate the wave speed, $v = \sqrt{\frac{T}{\mu}}$,for each of these three tensions. Find the wave length for each of these patterns ($\lambda = 2\lambda_{1/2}$). Then you can determine the frequency of oscillation from $f\lambda = v$. Compare your experimental frequency to the actual frequency of 60 Hz by using the % error. You will find these calculations on the spreadsheet for today's lab.

Experiment 2: Replace your fat cord with about 3-4 m of thread. I have a 20 meter section of this thread that you can weigh in order to obtain the mass per unit length. You'll need to ask for this. Connect the other end of the string to a weight hanger as shown. Once again, calculate the tension in your thread, and then calculate the velocity of a transverse wave from $v = \sqrt{\frac{T}{\mu}}$. Find your wavelength and then compare your experimental frequency to the actually oscillator frequency by using the % error. With a weight hanger and add weights to produce necessary tensions. **Note** that in the spreadsheet, you should enter a length of 20 m if you weight the mass of the 20 m long thread.



Experiment 3. In this portion of the lab, you will make direct measurements by timing the speed of a pulse and compare it to theoretical values. **As a big hint**, do not put too much tension on the cord because the pulse will travel faster than you can accurately time. You and your lab partner should find a nice long open area and take the cord with you. You will also need the long tape measure, a spring scale and a stop watch. The idea for this portion of the lab is to send a pulse down the string and time how long it takes for this to travel through a length 2L. Thus, you can obtain the wave speed directly. Your response time will, of course, greatly degrade measurement quality here but the point will still be made if you are careful enough. I recommend not applying very much tension to the cable since you will want the wave speed to be as low as possible here. On the spreadsheet, I have provided you with the mass of the long string, its unstretched length and also its outstretched linear mass density. When you stretch the line, this mass density will change: thus you will need to measure the stretched length. The mass, however will stay the same. You will need to use the entire length of the cord here in order to get a time which is significantly larger than your reaction time.

Conclusions and write-up: You should make sure that you completely understand the roles played by tension, mass per unit length, wavelength and frequency in determination of the frequency of a wave on a string. Your discussion may include a derivation of the frequencies for standing waves between two rigid boundaries. Ultimately, remember that the reason you are doing this lab is to understand these concepts so be sure that your writeup reflects you understanding (which is required, of course to be correct).

One thing that I'd like for you to see from this lab is that for harmonic waves on a long string, the wave length increases as tension (and thus wave speed) increases. This is explained by the following: imagine a pulse placed on the string. The higher the wave speed is, the further the wave can travel in the time of one oscillation of the oscillator.