

A wire has a length of 2 m and a total mass of 0.01 kg.

What tension must the wire be under in order to produce a fundamental mode of $f=100$ Hz when (a) two fixed boundaries exist and (b) one fixed and 1 free boundary exists.

Suppose a spring-mass system has a frequency of 1 Hz. How large must the spring constant be to produce this frequency with a 10 kg mass?

How long must a simple pendulum be at the surface of the earth to produce a frequency of 1 Hz?

A physical pendulum has a moment of inertia about one end given by $I=\beta mL^2$. If the pendulum is hinged about this end, find the frequency of small oscillations about equilibrium when $\beta=1.5$ and $L=1$ m.

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$$2 \text{ fixed BC: } f_1^f \lambda_1^f = v: \frac{1}{2} \lambda_1^f = L \Rightarrow \lambda_1^f = 2L: \Rightarrow f_1^f = \frac{v}{\lambda_1^f} = \frac{v}{2L}$$

$$2 \text{ free BC: } f_1^u \lambda_1^u = v: \frac{1}{4} \lambda_1^u = L \Rightarrow \lambda_1^u = 4L \Rightarrow f_1^u = \frac{v}{4L}$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow v^2 \mu = T$$

$$\text{fixed: } [2L f_1^f]^2 = \frac{T}{\mu} \Rightarrow T = [2L f_1^f]^2 \mu = [2 \times 2 \times 100]^2 \times 0.005 = 800 \text{ N (hang an 80 kg mass)}$$

$$\text{free: } [4L f_1^u]^2 = \frac{T}{\mu} \Rightarrow T = [4L f_1^u]^2 \mu = [4 \times 2 \times 100]^2 \times 0.005 = 3200 \text{ N (hang a 320 kg mass)}$$

Suppose a spring-mass system has a frequency of 1 Hz. How large must the spring constant be to produce this frequency with a 10 kg mass?

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f \Rightarrow k = [2\pi]^2 m = 395 \text{ N}$$

How long must a simple pendulum be at the surface of the earth to produce a frequency of 1 Hz?

$$\omega = \sqrt{\frac{g}{L}} = 2\pi f \Rightarrow g = [2\pi f]^2 L \Rightarrow L = \frac{g}{[2\pi f]^2} = \frac{9.8}{[2\pi]^2} = 0.248 \text{ m}$$

A physical pendulum has a moment of inertia about one end given by

$I = \beta mL^2$. If the pendulum is hinged about this end, find the frequency of small oscillations about equilibrium when $\beta=1.5$ and $L=1$ m.

$$\Gamma = \frac{L}{2} mg \sin \theta \approx \frac{-mgL\theta}{2} = I \frac{d^2\theta}{dt^2} \Rightarrow I \frac{d^2\theta}{dt^2} + \frac{mgL}{2} \theta = 0$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{mgL}{2I} \theta = 0 \Rightarrow \omega = \sqrt{\frac{mgL}{2\beta mL^2}} = \sqrt{\frac{g}{2\beta L}}$$

$$\omega = \sqrt{\frac{9.8}{2 \times 1.5 \times 1}} = 1.807 = 2\pi f \Rightarrow f = \frac{1.807}{2\pi} = 0.288 \text{ Hz}$$