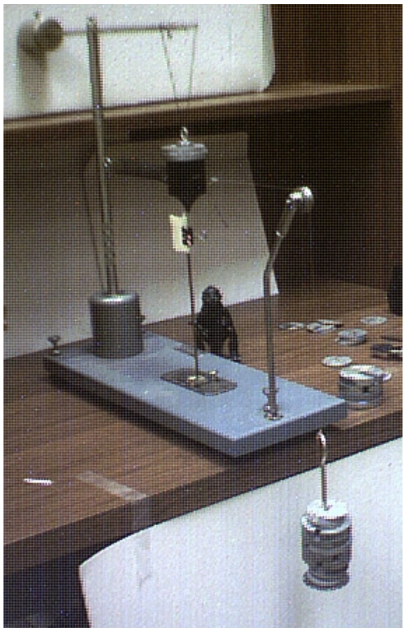
Centripetal Forces & Hooke's Law

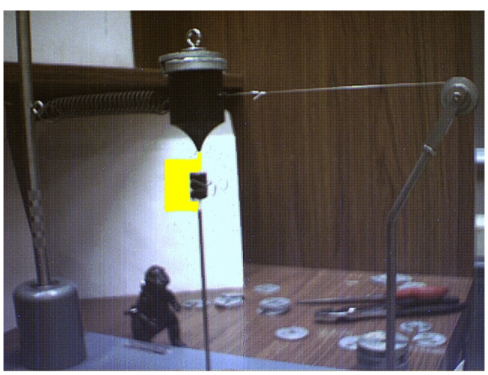


In today's lab, you will use the Welch Scientific Company's Centripetal Force Apparatus to verify the relationship between centripetal force, mass, velocity and radius. You may recall from class that for a mass moving in uniform circular motion, the centripetal force is given by:

$$F = m\frac{v^2}{R} \quad \text{(Equation 1)}$$

where F is the centripetal force, m is the mass undergoing circular motion, R is the radius of the circular orbit and v is the tangential velocity of mass m.

We will explore this for several different values of m in this lab, namely the mass of the pendulum. I will call this mass m_p. You will (by screwing masses onto the top of the pendulum) also investigate it for m_p, $m_p + 50g, m_p + 100g, m_p + 150g$. Experimental note: when you run your experiments, the upper most screw must be tight and the correct position for the pendulum is over top of the indicator bar with the spring, string, bar, and stand **making a square**. If these two things are not done, you will have larger experimental errors.



We could also vary the radius of orbit but this proves to be more difficult so we will keep R fixed in this lab. The image to the left shows the apparatus.

There is a small tab of paper attached to the upward sticking rod. When the apparatus is spinning at the correct velocity so that the pendulum mass is aligned over the metal indicator, and the mass is spinning, you will hear a click as the mass strikes the paper tab as shown in the image to the left.

Procedure: For each of the 4 masses, you should find the time T_n that it takes the system to spin through n revolutions. The period (T) is then given by

$$T = \frac{T_n}{n}$$

Here, n should be at least 20, but you can do more if you are so inclined. T is the amount of time for one revolution. Next, you should measure the distance from the center to the tab of paper. This will be R. Be sure to convert this measurement into meters. Now, in order to find out how much force is required to move the mass to the tab of paper, use the weight hanger together with the special attachment which I have provided (you can see this in the second figure). Add weight as shown in the first image until the pendulum is aligned with the tab of paper.

A question now: what are you measuring when you measure this particular force which is given by:

$$F = mg\,?$$

The answer is that you are measuring how much force the spring is exerting on the pendulum while it is swinging trying to pull the mass backward towards the center. Of course, you're not going to want to be spinning the thing until you remove that paperclip attachment that I've made for you!

Now the next thing is this: How do you know what the tangential velocity is that you are measuring? And here is the answer: in a period T, the mass travels through a distance given by:

$$s = 2\pi R$$

Thus you are able to fine the tangential velocity pretty easily:

$$V = \frac{2\pi R}{T}$$

Important experimental notes

Here are important notes to reduce your errors: when the spring is removed from the pendulum, the pendulum should hang right over the rod that is sticking up. You will need to adjust the screw on top to make sure this is the case. Failure to do this will produce larger errors. After reconnecting the spring to the pendulum, you will want to balance the system by moving the adjustable mass on top. The system is balanced when it rotates without jumping around. You will also want to make sure the system is leveled by adjusting the legs on the board.

Analysis: The centripetal force is related to the mass, velocity and radius by the first equation above. For each of your measurements, calculate the experimental centripetal force from Equation 1. Compare these calculations to your measured centripetal force by using percent error. You may consider that the correct value for the centripetal force is obtained by the masses attached to the hanger.

In this lab, you will also want to plot a graph of m vs. $(1/v^2)$ Looking at the theoretical equation for the centripetal force, you would expect the slope of this graph to be equal to

$$\text{slope} = \frac{1}{RF}$$

This is how this comes about: you measure m, v and R. Using the first equation, you can then obtain:

$$F = m\frac{v^2}{R} \Rightarrow \frac{1}{v^2} = \frac{m}{RF}$$

In this experiment, you are varying m and observing the effect that this has upon v. Now let's define a new variable, y such that:

$$y \equiv \frac{1}{v^2}$$

In terms of this variable, the equation looks like:

$$y = \left[\frac{1}{RF}\right]m = [\text{slope}]m$$

So if you plot a graph of m vs. y, the slope of this graph will be

$$\text{slope} = \frac{1}{RF}$$

You might ask: why worry about making this linear? One easy answer is this makes the fitting process simpler. Now once the slope has been obtained, you can determine the force from:

$$F = \frac{1}{R[\text{slope}]}$$

Compare your slope to the theoretical value of F that is measured by hanging weights over the end of it as shown in the second figure. In principle, you should obtain fairly good agreement here. Discuss in your write-up where experimental errors may have entered into the results.

Part II: Hooke's Law

Important experimental note

You will want to make sure your springs are in the correct packages labeled 632-050, 632-051 and 632-052. The spring with the lowest spring constant is 632-050 while the highest is 632-052. An easy to determine It is worth while to note that the "50" springs ought to have a black mark on them. I have a tester to make sure you have the right spring also. The spring with the most stretch is 632-050 while the spring with the least stretch is 632-051. Make sure after your experiments that you replace the springs into the correct package.

Each of these springs are part of a Hooke's law setup and the nice thing about these springs is that they all have the same diameter (7 mm) and the same length (55 mm). The expected spring constants for the individual springs are:

Spring number	spring constant [N/m] expected
632-050	5
632-051	8
632-052	70

You will need to enter these as the expected values in the calculator.

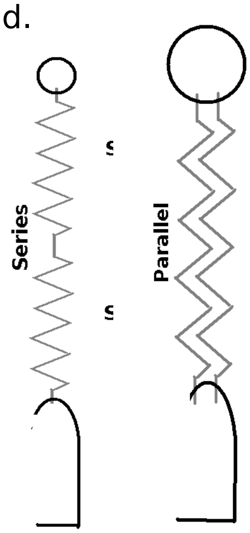
part 1: measurement of the spring constant of an individual spring.

Arrange your spring holder and the metal ruler so that the larger numbers of the ruler are furthest away from the spring holder. Attach a 5 g mass holder to the free end of the spring. Place on the holder 10g . Measure the bottom of the weight holder (which is referenced to an arbitrary position).

Place 5 g additional mass on the mass holder (for the 70 N/m spring, place an additional 50 g (or so) on the holder) and measure the position of the bottom of the mass holder. Do this until you have about 75 g (you will need about 400 g total on 632-052) total on the spring but do not! overstretch the springs (do not exceed more than about 12 cm total stretch of the springs). You should record your data for total mass (g) vs. position (cm) in a table. Note that you need to measure the “mm” side of the metal ruler and that the “mm” refers to the smallest unit on the ruler. Luckily, in the case of the curved rulers, the larger numbers are on the flat end of the ruler for metric readings.

For each of the 3 springs, use the Hooke’s law spreadsheet to find the spring constant from the slope of the graph and the % error.

Now I want you to do two additional experiments with two of your springs from the 632-050 packet. As a result of this part of the lab, you will be able to answer from your data how spring constants in parallel vs series add. The two diagrams below show the two spring configurations of which I want you to find the spring constant. You can use the same spreadsheet here but the expected values here are not valid.



The spring constants should add in one of the following ways:

$$k = k_1 + k_2 \text{ or } \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}.$$

Since the spring constants are the same for each spring, this is tested by simply seeing if the configuration doubles the spring constant of reduces it by ½. In your writeup, you should answer this question and provide the evidence from your data to support your conclusion. You will need to use the spreadsheet to calculate your spring constants again although the accepted value at the time you do the calculation will be unknown.

Part III: Slinky investigations

Not every spring obeys Hooke’s law. Part of this may be due to that material that the spring is constructed of and part of it may be due simply to the weight of the spring. In this final part of the lab, I want you to investigate a slinky. In particular, you should weight the slinky, and then count the number of turns on the slinky. Holding the slinky up by a piece of paperboard, increase the number of turns of the slinky below the paperboard, measuring also the length of the slinky below the paperboard. Plot these results on the Slinky spreadsheet calculator. Other than a discussion of how the length of the slinky varies with number of turns here (which is proportional to the mass the spring is supporting) no further analysis is required.